

## Abstract

The motion of a magnet falling through an aluminium pipe with terminal velocity was modeled by assuming the magnetic braking force to be proportional to the velocity of the magnet. The predicted time of fall from the simulation matched the measurements for a wide range of weights for the magnet. The simulation was then used to answer questions such as - how do terminal velocity, time taken to attain the terminal velocity, and the distance fallen by the magnet before attaining terminal velocity depend on the weight of the magnet? The Vpython simulation was thus used as a virtual lab to conduct virtual experiments to answer new questions about the modeled phenomenon, which may have otherwise been rather difficult for a high school student due to lack of the required mathematical skills, apparatus or even time to conduct the experiments.

## Introduction

When a powerful neodymium cylindrical magnet is dropped inside a vertical aluminium cylinder, it experiences a 'magnetic braking' effect causing it to quickly attain terminal velocity and emerge out of the cylinder much later than expected. This is a fun and easy demonstration that never fails to captivate the students' interest and can be performed to introduce Faradays' Laws of Electromagnetic Induction, Lenz's law and the effects of eddy currents.

As the magnet falls through the aluminium cylinder, it induces circular eddy currents in the 'aluminium rings' making up the aluminium pipe, which in turn induces an opposing magnetic field that retards the motion of the falling magnet.<sup>1</sup>

While the detailed mathematical treatment<sup>2-5</sup> seems very daunting for a high school student, it is remarkably easy to make a simulation in Vpython<sup>6</sup> to model the magnetic drag force accurately and verify the predictions of the simulation by direct measurements using a high-speed camera<sup>7</sup> (found in most high-end cellphones like the iPhone) to film the motion of the magnet as it falls through the metal pipe.

It is possible to track the motion of the magnet inside the opaque metal pipe by tracking the motion of a mass attached to the magnet with a long string such that the mass hangs outside the bottom end of the vertical pipe. Analyzing the video using TRACKER<sup>8</sup>, we can accurately observe the changes in position with time to answer questions about the motion like

1. How does the position and velocity of the magnet vary with time?
2. How does the total time taken to fall through a non-magnetic metal pipe of given length vary with the weight of the magnet?
3. How does the terminal velocity attained by the magnet vary with the weight of the magnet?
4. How does the time taken to attain terminal velocity vary with the weight of the magnet?
5. How does the distance fallen by the magnet before attaining terminal velocity depend on the weight of the magnet?

## The Model

$$F_{net} = m \frac{dv}{dt} = mg + F_d = mg - bv$$

Where  $b$  = magnetic drag coefficient (determined by trial and error)

Solving analytically, we get

$$v(t) = \frac{dv}{dt} = v_T \left[ \left( 1 - e^{-\frac{t}{\tau}} \right) \right] \quad \text{and} \quad y(t) = v_T \tau \left[ \frac{t}{\tau} - 1 + e^{-\frac{t}{\tau}} \right]$$

where terminal velocity  $v_T = \frac{mg}{b}$  and time constant  $\tau = \frac{m}{b}$ .

### Initialisation

$$dt = 0.001 \text{ s}$$

$$b = 4.2 \frac{\text{N}}{\text{m s}^{-1}}$$

$$\vec{w} = -mg\hat{j}$$

$$\vec{F}_d = -bv\hat{j}$$

$$\vec{F}_{net} = \vec{w} + \vec{F}_d$$

$$\vec{v} = \vec{v} + \frac{\vec{F}_{net}}{m} dt$$

$$\vec{r} = \vec{r} + \vec{v} dt$$

### Exit Criteria for the Loop

To find the time of fall through the aluminium tube while magnet.pos.y > -0.253:

To find terminal velocity and also the time taken and distance travelled to attain the terminal velocity while abs(Fnet.y) > 0.0001:

DataWriter.writerow([t, magnet.pos.y, magnet.vel.y, Fd.y, Fnet.y])  
Thus, the last row of the data in the .CSV file will contain the required values.

## Methods and Materials

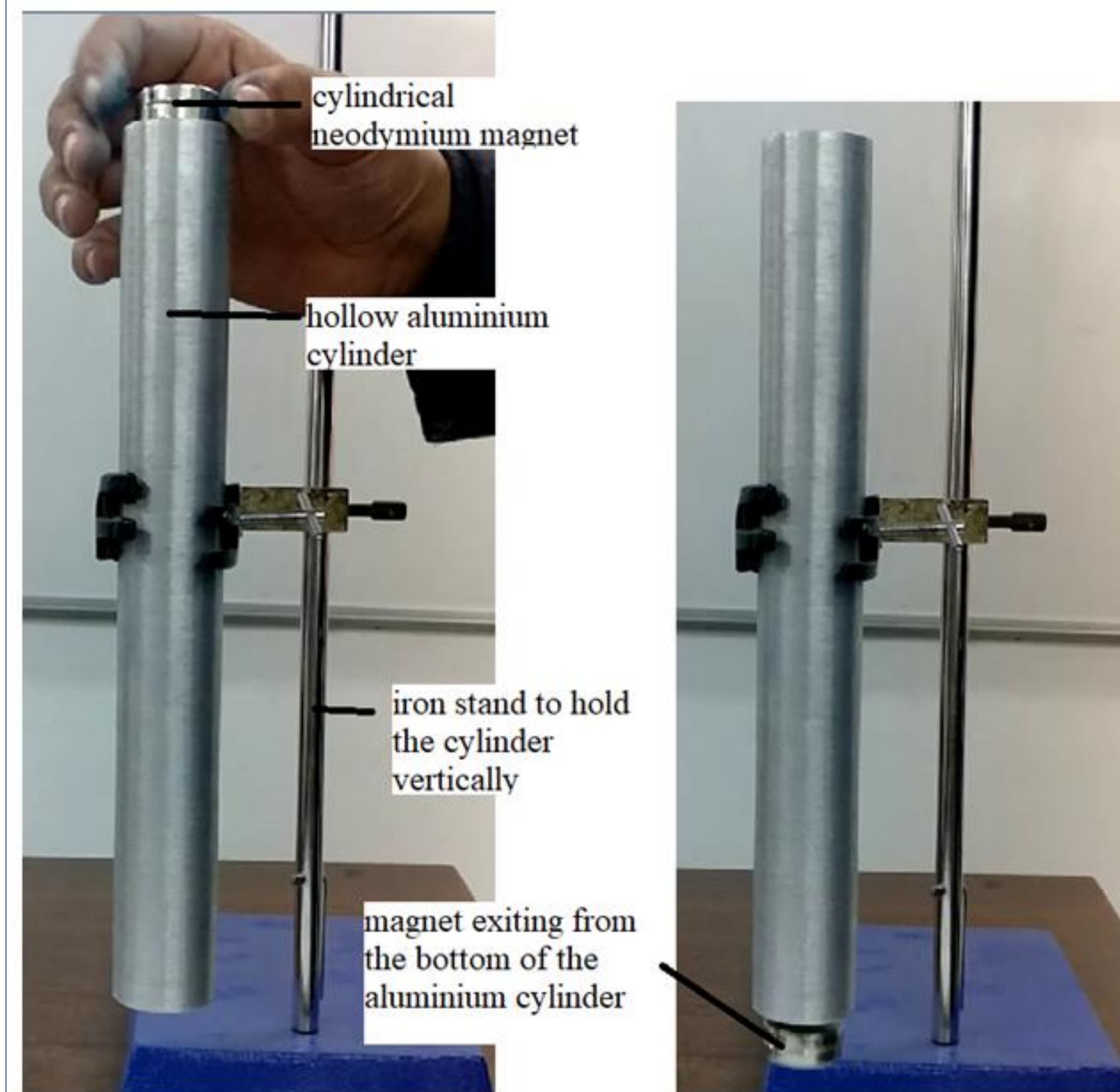


Figure 1 Measuring time of fall of magnet through aluminium tube

## Simulation Outputs – real-time Graphs

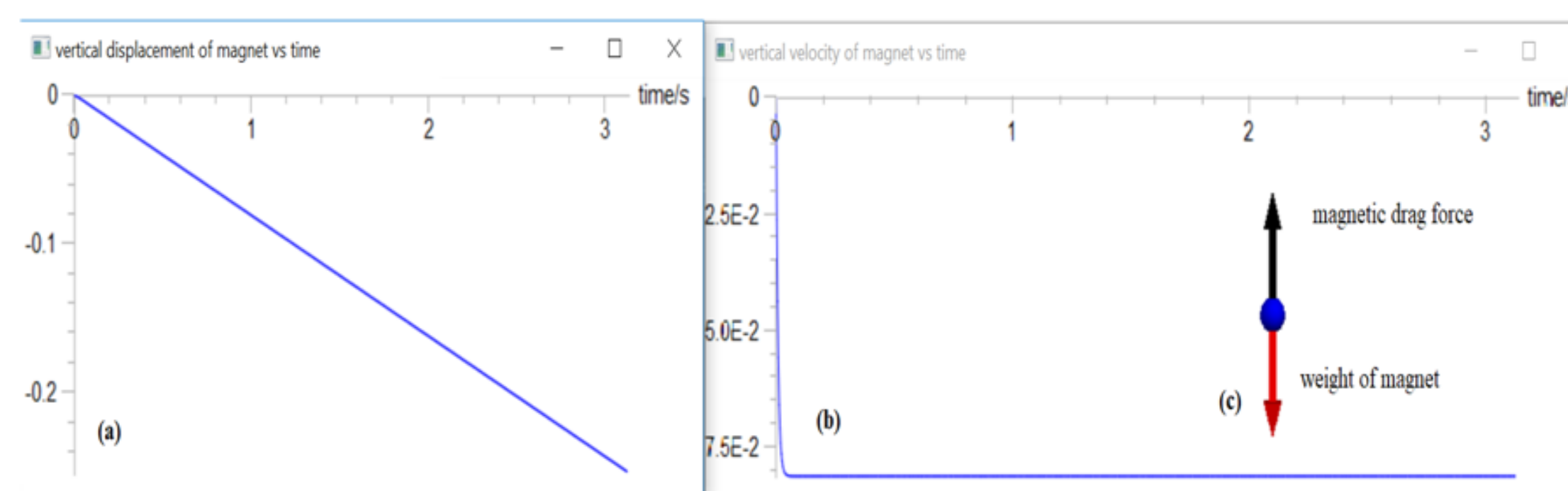


Figure 3 Screenshot of the simulation output windows for 0.0341 N magnet: (a) vertical displacement vs time graph (b) vertical velocity vs time graph (c) real-time free-body diagram of magnet

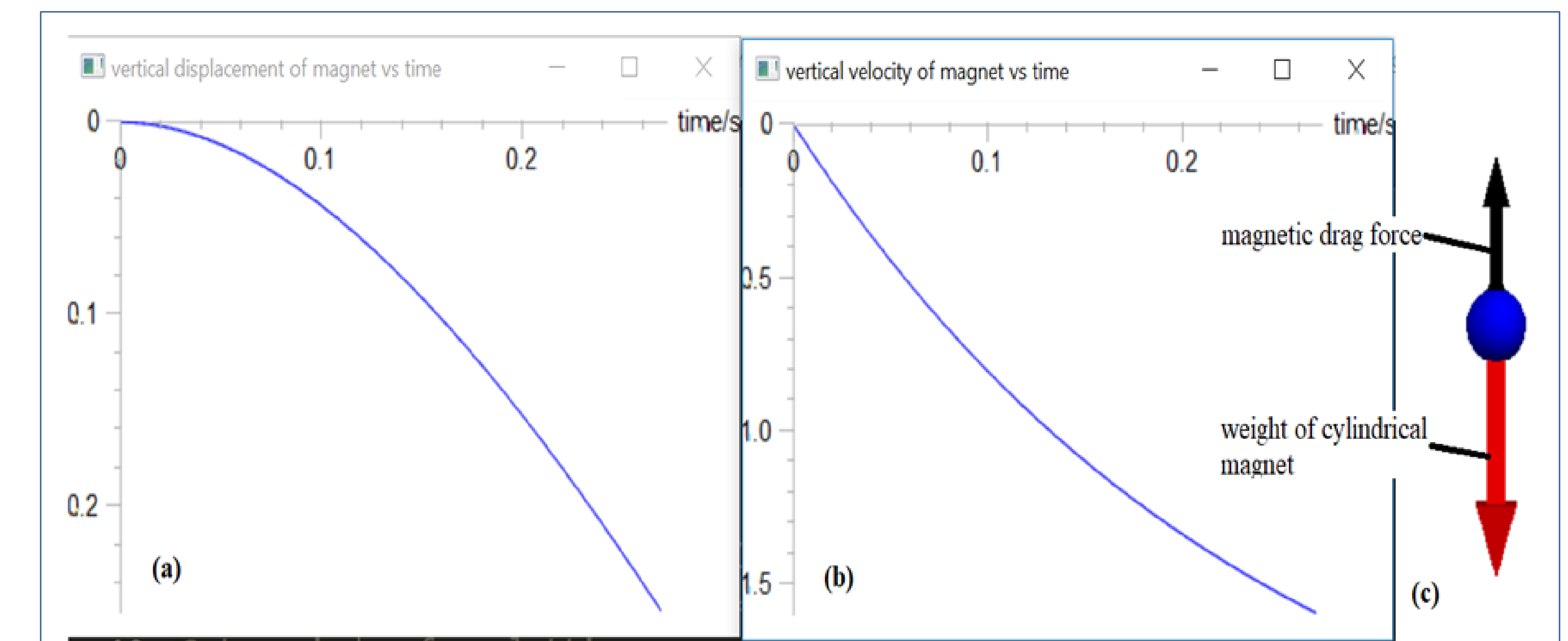


Figure 4 Screenshot of simulation output for magnet weight = 9.944 N: (a) vertical displacement-time graph (b) vertical velocity-time graph (c) free-body diagram of falling magnet

## Results and Discussion

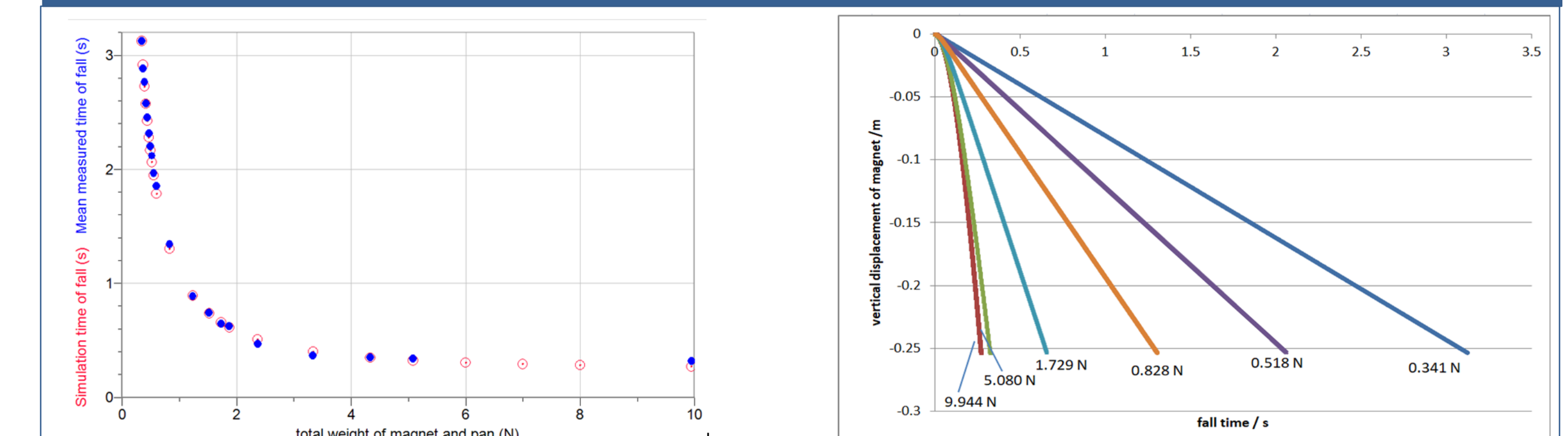


Figure 5 Comparison of time of fall of magnet through the 25.3 cm long aluminium tube as measured by experiment and predicted by simulation

Figure 6 The displacement-time graphs predicted by the simulation were found to be in perfect agreement with the observed motion in the videos

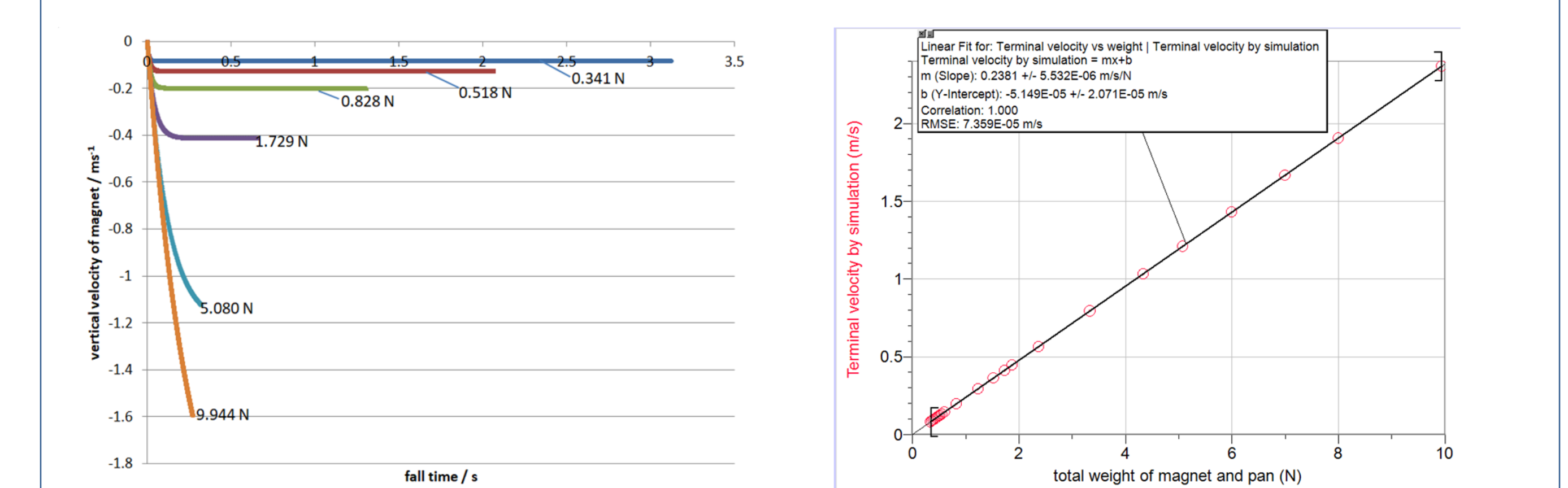


Figure 7 The velocity-time graphs predicted by the simulation were found to be in perfect agreement with the motion observed in the videos.

Figure 8 Simulation predicts the Terminal velocity to increase linearly with weight of the magnet

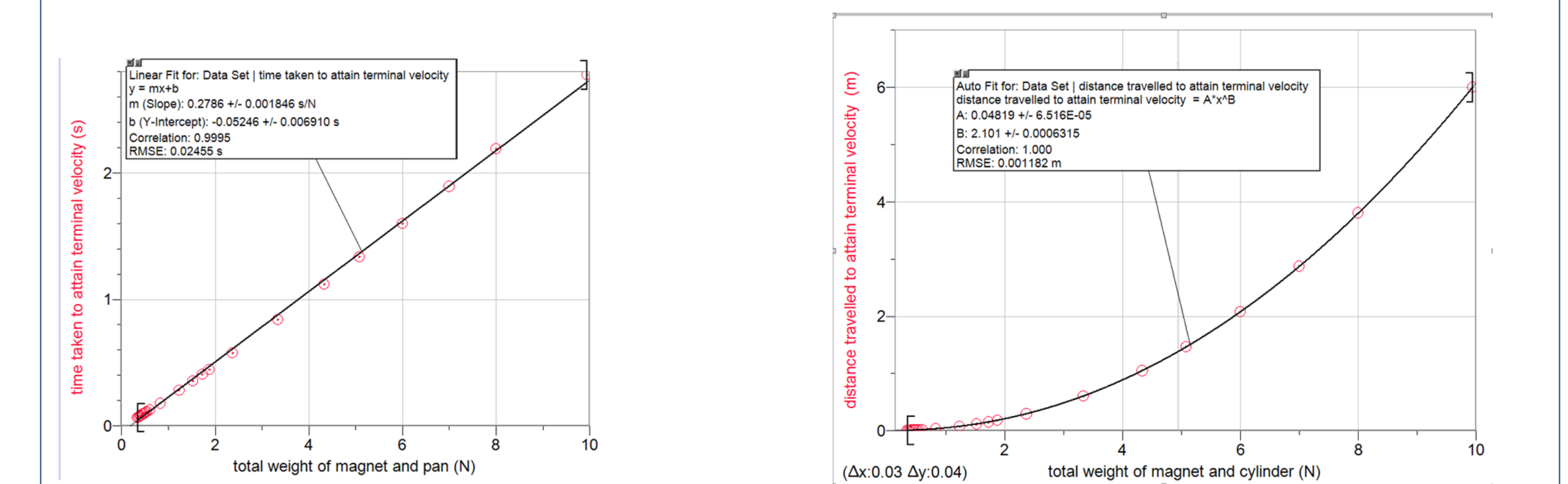


Figure 9 Simulation predicts the time taken to attain terminal velocity to vary linearly with the weight of the magnet

Figure 10 Simulation predicts the distance travelled by the magnet before attaining terminal velocity to vary as the square of the weight of the magnet

## Conclusion

The motion of a cylindrical magnet falling through an aluminium tube can be modelled accurately by assuming the magnetic drag force to be proportional to the velocity of the magnet and opposite in direction. A simulation can be made easily in Vpython to solve the differential equation derived from the net force equation. It can be run multiple times to generate the position vs time data for the motion of the magnet which can then be used to answer a variety of questions about the motion.

## Contact

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## References

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