Coefficient of Rolling Friction and Pressure Inside a Football

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Abstract

A football at different internal pressures was rolled from rest down an inclined smooth wooden plank. The acceleration of its centre of mass was measured by video analysis using TRACKER software. It was found to increase with excess pressure inside the football until it reached a constant value at an excess pressure of 36.43 kPa and above corresponding to a minimum constant value of rolling resistance. The decrease in the acceleration at lower pressures was linked to the increase in contact area of the football, which resulted in greater hysteresis energy loss due to the deformation of the football while rolling. This was modelled by defining rolling resistance coefficient δ as the offset distance of the line of action of the normal reaction producing a retarding torque on the football. δ was found to be an inverse exponent function of excess pressure.

Introduction

Research Question: **How does the rolling resistance of a football rolling down an inclined wooden surface depend on the excess air pressure inside it?**

Rolling resistance is defined[3] as the opposing torque acting on the football due to the normal reaction and the perpendicular distance between the line of action of the normal force and the axis of rotation passing through the center of mass of the football. The moment arm of the normal reaction, which is simply the radius of the circular area of contact of the football with the inclined surface, is defined as the *coefficient of rolling resistance(δ).*

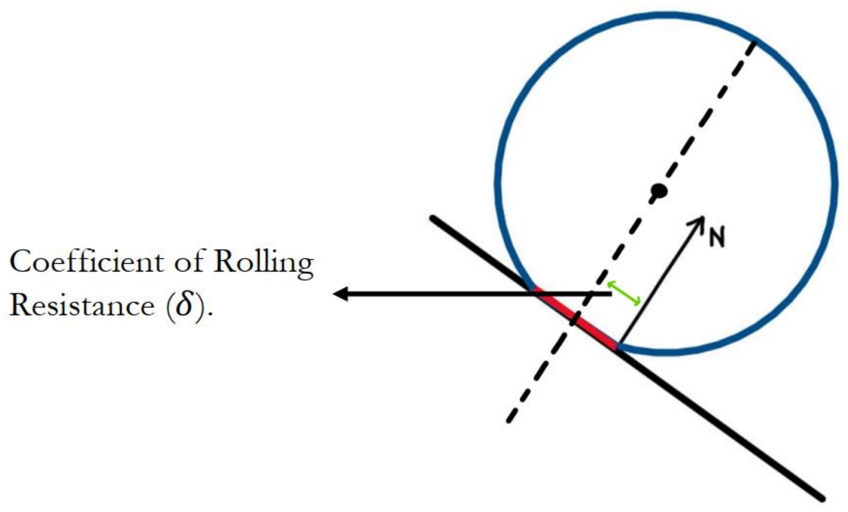

Figure 1. Defining the coefficient of rolling resistance

Hypothesis: By measuring the acceleration of center of mass of the football, we can compute the retarding torque experienced by the football and thus find its rolling resistance. Since the acceleration of the center of mass of the football is dependent on its excess pressure, we can find out how the rolling resistance experienced by the football is related to the excess air pressure.

When excess pressure increases to a certain value, the football will assume the shape of a perfectly rigid spherical shell, and would thus have maximum acceleration. Any further increase in the excess pressure would not change the shape of the football anymore, thus keeping the acceleration of the center of mass constant for higher excess pressure values. Thus, we expect that as the excess pressure inside the football increases, the acceleration of its center of mass increases asymptotically, becoming constant at a particular value.

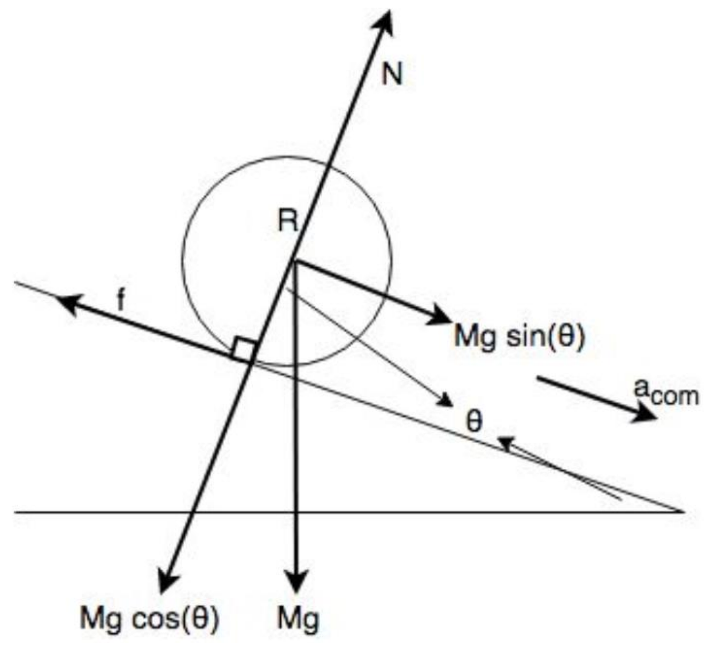

Figure 2. Free Body Diagram of the rolling football.

Using the net force equation, the maximum acceleration which the football can acquire by approaching the shape of a perfectly rigid spherical shell was derived to be $a_{com} = \frac{3}{5} g \sin(\theta)$ = 0.61 ms^{-2}.

Thus, the graph of a_{com} of football vs excess pressure should asymptotically reach this value, which can be modelled using a Logistics differential equation as

$$\frac{d(a_{com})}{d(P_e)} = G a_{com}\left(1 - \frac{a_{com}}{a_{max}}\right) \text{ ; where G = growth rate.}$$

Solving, we get $a_{com} = \frac{0.6 \sin(\theta)}{1 + 0.366e^{-0.075P_e}}$ which was found to agree beautifully with the experimental data as shown in Figure 5.

Methods and Materials

Figure 3. Vernier pressure sensor was used to measure the air pressure inside the football, which was varied using a motor pump before rolling down the inclined plane.

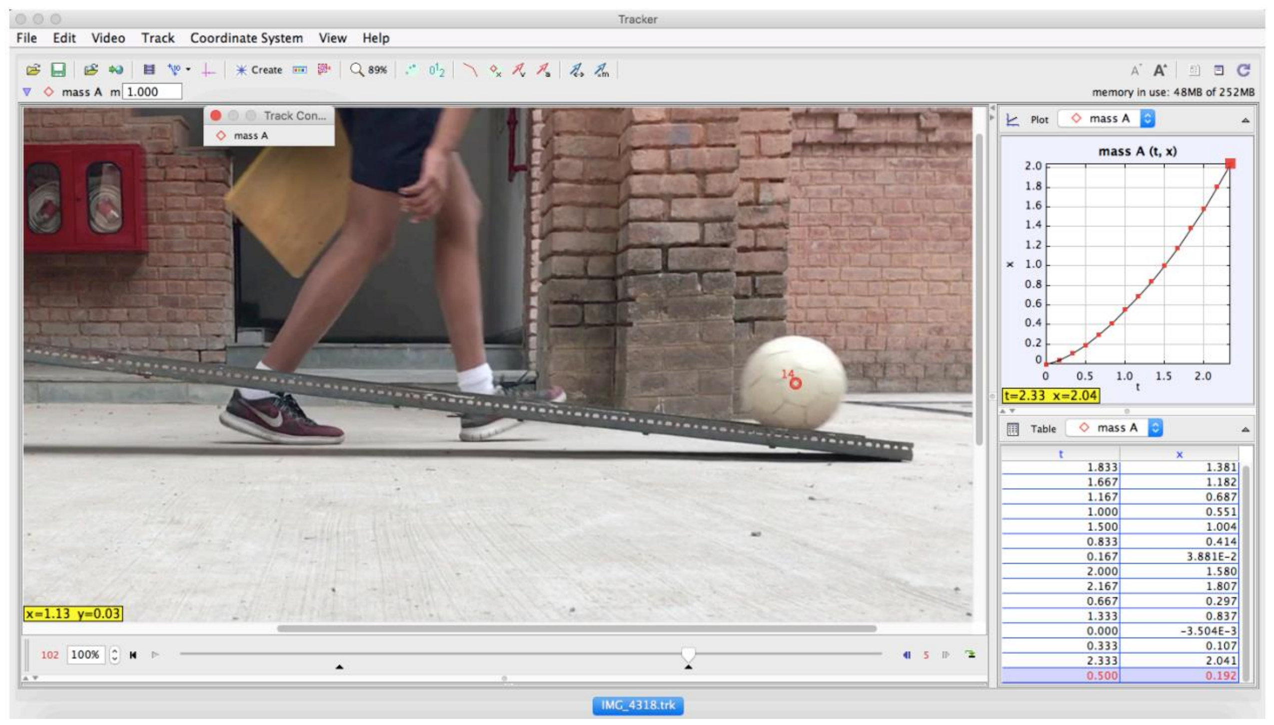

Figure 4. The measured acceleration was computed from the quadratic fit of the Position-time graph of the football using TRACKER.

Results and Discussions

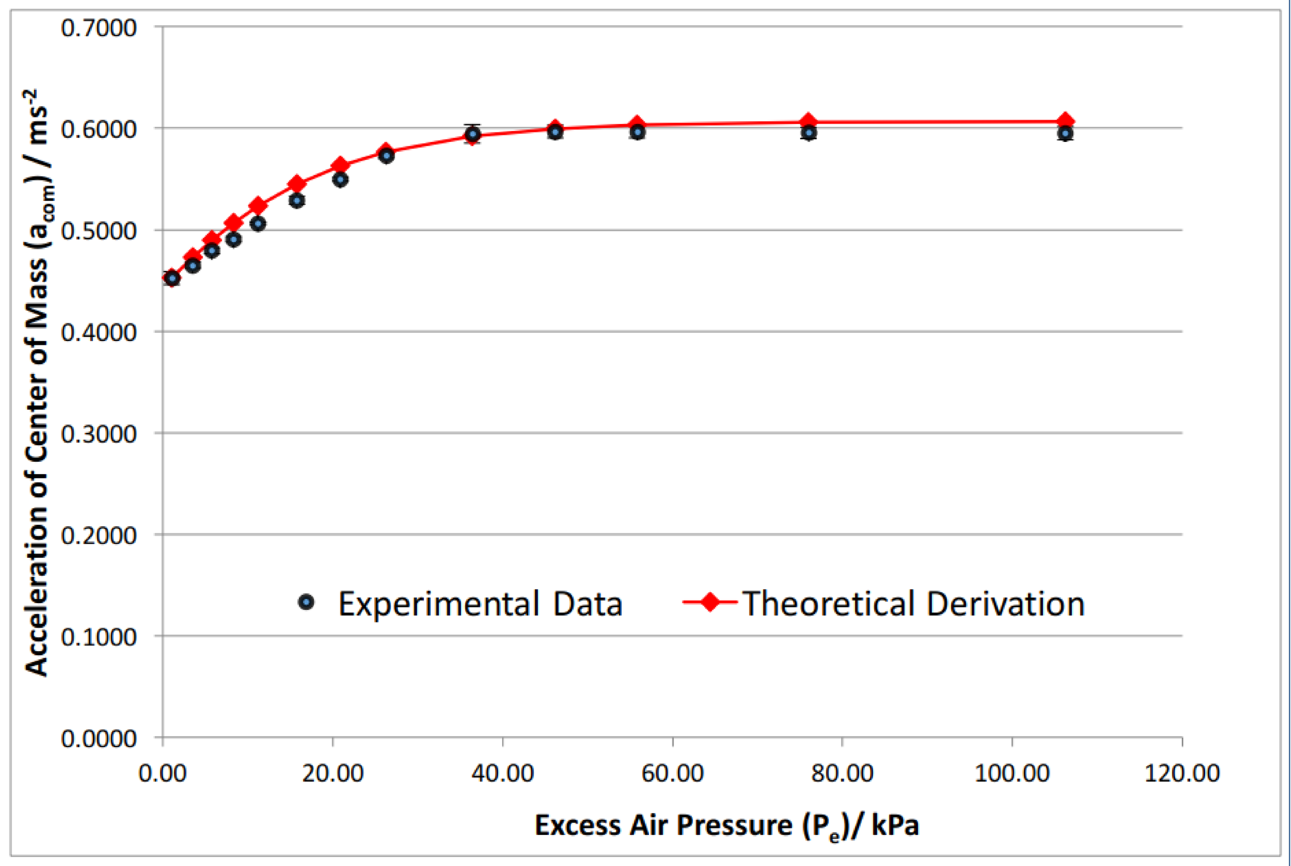

Figure 5. Logistic function gives a perfect fit for the experimental data which also Verifies the value of the maximum acceleration predicted to be 0.61 ms^{-2}.

Coefficient of rolling resistance was calculated from the measured a_{com} of the football using $\delta = r(\tan(\theta) - \frac{5}{3}\frac{a_{com}}{g\cos(\theta)})$ and was found to agree beautifully with the δ predicted using the Logistics function which is $\boldsymbol{\delta = r\tan(\theta)\left(1 - \frac{1}{1 + 0.366e^{-0.075P_e}}\right)}$ as shown in Figure 6.

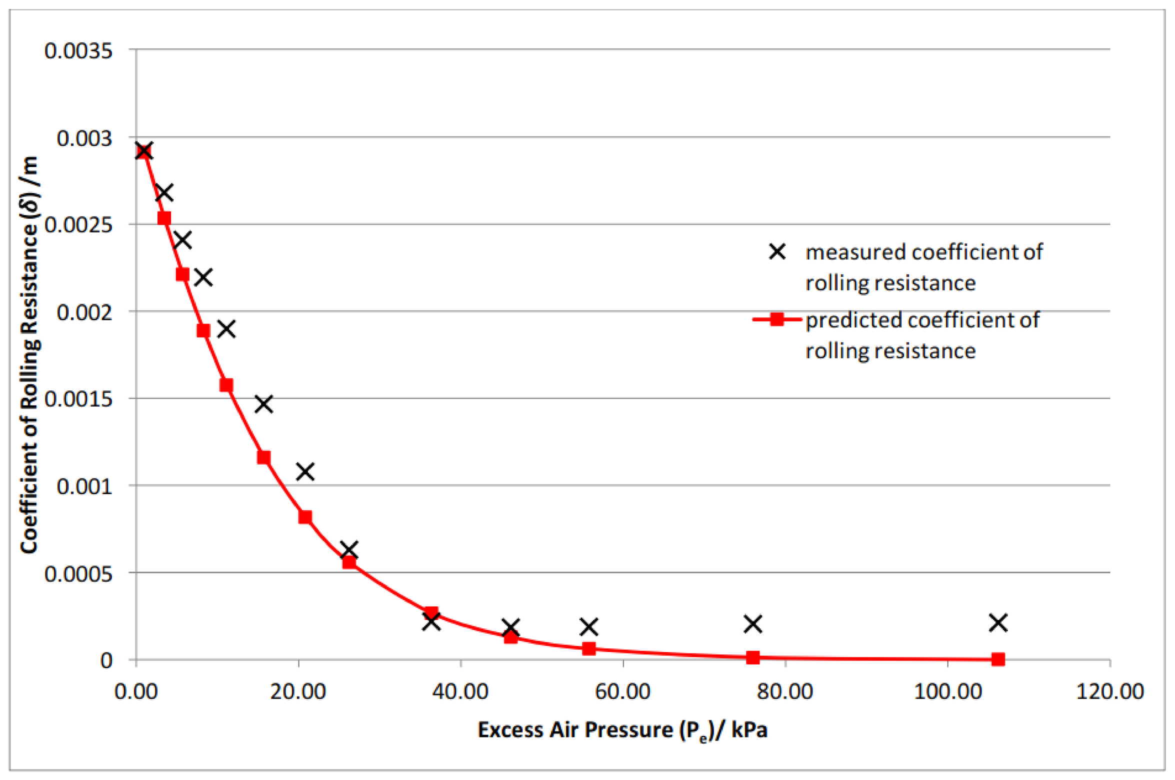

Figure 6. Measured values of coefficient of rolling resistance agrees with the values predicted using the Logistic function.

Conclusions

An inverse asymptotic relationship exists between the excess air pressure inside a football and the rolling resistance experienced by it. Thus, as the excess air pressure inside the football increases, it starts assuming the shape of a perfectly rigid spherical shell and the rolling resistance experienced by it decreases asymptotically to a constant value just more than zero. This was shown by the experimental data obtained, and also proved by the derivation of the equation $\boldsymbol{\delta = r\tan(\theta)\left(1 - \frac{1}{1 + 0.366e^{-0.075P_e}}\right)}$

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